Parameter Estimation for Physics-Informed Penalized Regression Models

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Spatial statistics plays a crucial role in the analysis of demographics (such as health, justice, and labor), economies (including trade and agriculture), and environmental factors (atmospheric and oceanographic). The variable of interest is often observed at a limited set of geographic locations, obtained through non-systematic sampling. These observations are dispersed within spatial domains characterized by irregular shapes and, at times, interior holes. Consequently, modeling approaches must address the intricate auto-correlation among these observations, the irregularity in sampling patterns, and the polygonal nature of the domains.

Linear partial differential equations (PDEs) are versatile models for capturing intricate spatial autocorrelation within irregularly shaped domains. However, parameter estimation for PDEs presents a challenging task. To tackle this issue, we propose a penalized regression framework that capitalizes on partially observed and noisy measurements to estimate and quantify the uncertainty of the PDE parameters and the subsequent PDE solution. Through simulations, we establish that our proposed approach surpasses current methodologies, attaining a three-fold enhancement in the accuracy of the parameter estimates.